Ph.D. Topic – 3IA Côte d’Azur

Mesh-based Shape Approximation with Guaranteed Progress Toward Global Optima

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Background. Several key developments have shaped this research direction:

The optimal approximation of 3D shapes using meshes is known to be an NP-hard problem. This means that finding the best possible mesh representation for a given shape—while minimizing the approximation error—is computationally challenging, as no polynomial-time algorithm can solve it optimally in all cases. Due to this complexity, researchers typically rely on heuristics, approximation algorithms, and optimization techniques to generate practical solutions that only reach local minima. Common approaches include:

- Iterative refinement methods that improve the approximation over time.
- Energy-based optimization that balances accuracy and mesh complexity.
- Graph-based techniques that seek near-optimal connectivity structures for mesh representation.

Recent advances in machine learning and data-intensive approaches have facilitated the search for better solutions, including global minima, using methods such as evolutionary computation and reinforcement learning.

Scientific Objective.

The primary objective of this Ph.D. thesis is to explore novel optimization algorithms for mesh-based approximation of 3D shapes. A specific challenge is to develop algorithms that are guaranteed to consistently progress toward the global optimum.

In this context, the optimum is defined in two ways:

- Minimal mesh complexity for a given error tolerance.

- Minimal approximation error for a given mesh complexity.

Most greedy algorithms employ:

- Local operators (e.g., edge flip, edge collapse, vertex insertion [6], vertex relocation).
- Variational approaches [5].
- Multi-stage strategies, separating topology optimization from geometry optimization [6].
- Expanded operator repertoires that integrate various transformation techniques [9].

Mesh algorithms—including meshing, remeshing, and mesh processing [1, 2] — typically utilize operators at different scales:

- Local (e.g., edge flip, edge collapse, vertex insertion, or relocation).
- Semi-local (e.g., sequences of edge flips or vertex relocations within a stencil).
- Global (e.g., relocation of all vertices simultaneously).

Each operator is associated with:

- A computational or energy cost.
- A benefit regarding the primary objective function (e.g., minimizing approximation error).

A crucial question is how to design an algorithm that organizes operators into a sequence where:

- The best operators (low cost, high benefit) are executed first to achieve an optimal tradeoff between time (or energy) and the objective function.
- The algorithm consistently progresses toward the global optimum.

A major challenge lies in the fact that measuring the benefit of an operator is often as costly as applying it.

Potential Research Direction. One promising approach is to use reinforcement learning or advanced machine learning to predict operator benefits and optimize their sequence. The same principle applies to ill-posed problems in geometric modeling and processing, where the objective function must be balanced across multiple criteria rather than focusing on a single optimization goal.

References

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