Physics Informed Neural Networks (PINN) for PDE solving based on the simulation of a ray tracing method

Riccardo Di Dio

Clotilde Djuikem

Gianluigi Lopardo

Nicolas Rosset

Tong Zhao

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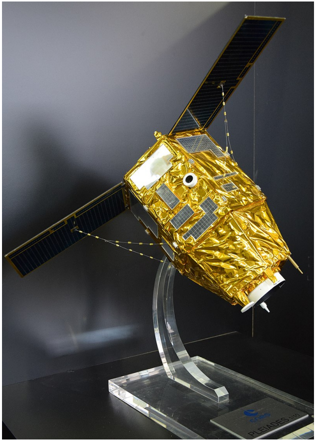
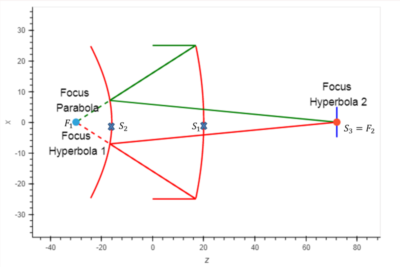
Context

Artificial satellites



17th century: approach 1



- Based on conic surfaces: parabola, hyperbola, ellipses
- Exact resolution
- Famous examples: Newton telescope (1 parabola)

Context

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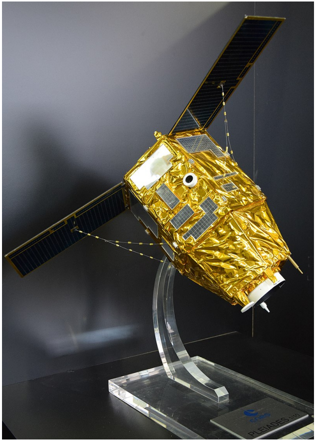
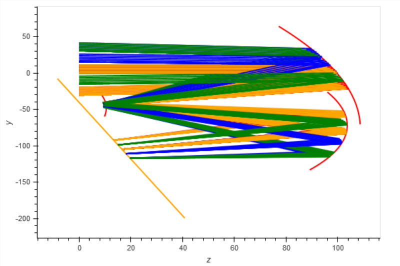


20th century: approach 2



- Use of freeform surfaces orthogonal polynomials or NURBS
- Parametric optimization problem
- Famous example: Three Mirrors Anastigmat (Gaia mission)

Problem

State-of-the-art

- Classical numerical methods such as Finite Elements
- Discrete approximation through statistical approach

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- Needs for alternative method
- Approximation of light intensity f
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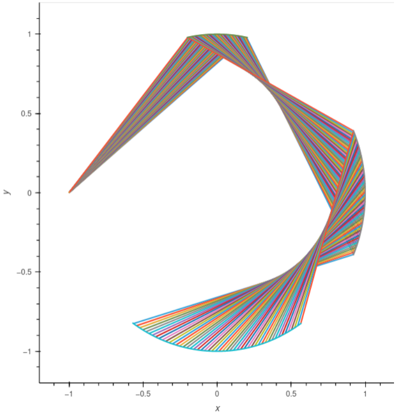
Work approaches

- Part 1: Statical approach
- Part 2: PINN approach

Part 1: Statistical methods

Modelling

Assuming that we know the light position p_0 and directions $\{d_1, \ldots, d_N\}$ of N rays, the mirror locates at the unit circle around the origin, we evaluate the light intensity on the mirror $f(p, d_i), p \in \mathbb{R}^2 \cap mirror, d \in S^1$, using a statistical approach



Part 1: Statistical methods

The light intensity is define by:

$$f(p, d) = \lim_{N \to \infty} f^N(p, d)$$

With

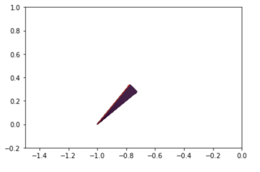
$$f^N(p, d) = \frac{1}{N} \sum_i \delta_{p_i, d_i}(p, d)$$

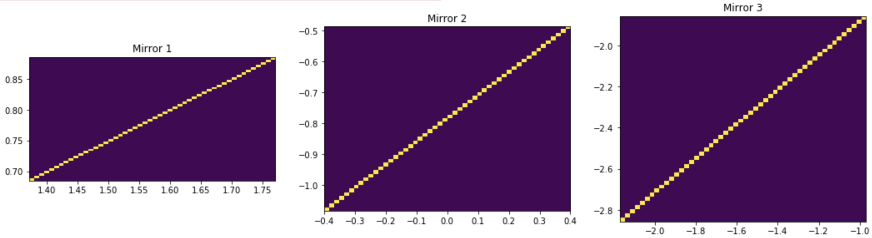
Question?

Make the histogram for the parameter pair (θ, d), where θ is a parameterization of the mirror (S^1 in our case) and d the directions of the lights.

Single source on unit circle

- Source position: $p_0 = (-1, 0)$
- Direction: $\pi/4 - 0.1 < d < \pi/4 + 0.1$
- Number of rays: 1000



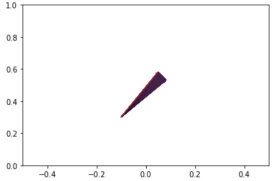


Remark 1

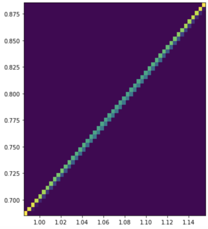
When the light source locates on the unit circle, the histogram is always uniform along the diagonals.

Single source inside unit circle

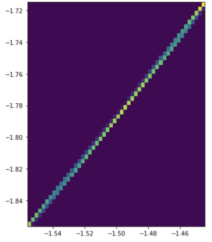
- Source position: $p_0 = (-0.1, 0.3)$
- Direction: $\pi/4 - 0.1 < d < \pi/4 + 0.1$
- Number of rays: 1000



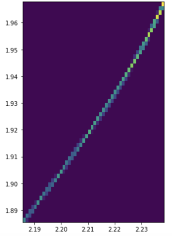
Mirror 1



Mirror 2



Mirror 3

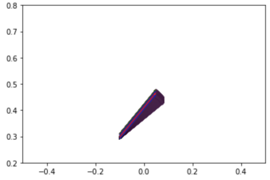


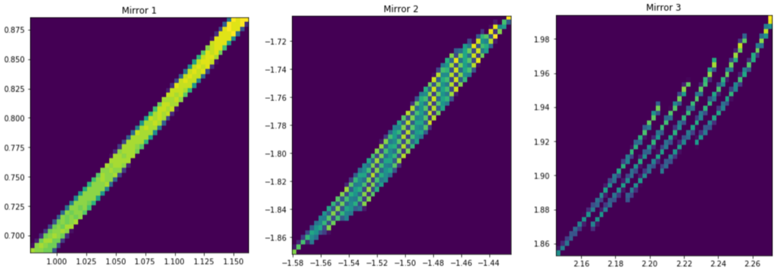
Remark 2

What happens if we have multiple sources?

Multiple sources on $x = -0.1$

- $p_0^y \in \{0.29, 0.295, 0.3, 0.305, 0.31\}$
- Direction: $\pi/4 - 0.1 < d < \pi/4 + 0.1$
- Number of rays: 5000



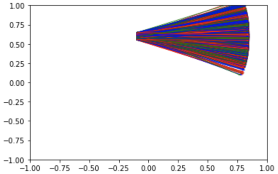


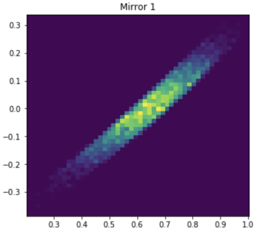
Remark 3

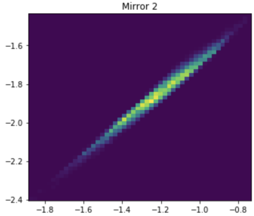
What happens if the source is no longer fixed?

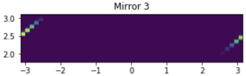
Uniform sources on $x = -0.1$

- $p_0^y = Uniform(0.29, 0.31)$
- Direction: $\pi/4 - 0.1 < d < \pi/4 + 0.1$
- Number of rays: 10000



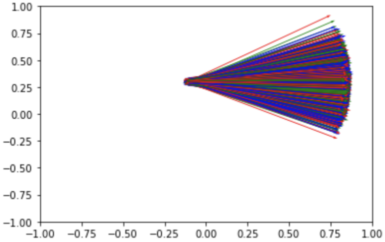


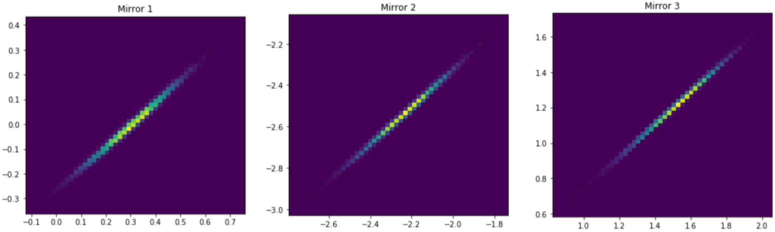




Gaussian source with small variance

- Source mean position: $(-0.1, 0.3)$
- Source variance matrix: $[[0.0001, 0][0, 0.0001]]$
- Light mean: 0
- Light standard deviation: 0.1
- Number of rays: 10000





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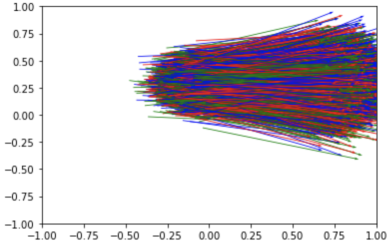

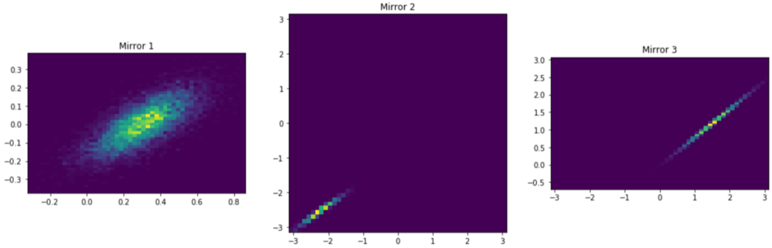

Part 2: PINN Approach

Modeling

We suppose that the equation is time independent and the density function f satisfies the following simplified version of a Liouville-Boltzmann equation:

$$\begin{cases} d \cdot \nabla_p f + \lambda f = 0\,, \quad \text{on}\,\Omega\,,\ 0 < \lambda << 1\,, \\ \qquad f(p,d) = f(p, d - 2(d,n)n))\,, \quad \text{on}\,\Gamma_-^M\,, \\ \qquad f(p,d) = \mathbf{G}_{\sigma_p}(||p - p_0||) \times \mathbf{G}_{\sigma_d}(||d - d_0||)\,, \quad \text{on}\,\Gamma_-^s\,. \end{cases}$$

where $(p,d) \in \Omega = \mathbf{D} \backslash \bar{W}_n \times \mathcal{S}^1$ and

$$\begin{cases} \Gamma^+ = \{(p,d) \in \Omega\,,\ n(p) \cdot d \geq 0\}\,, \\ \Gamma^- = \{(p,d) \in \Omega\,,\ n(p) \cdot d < 0\}\,, \\ \Gamma_-^M = \{(p,d) \in \Gamma^-\,,\ p \in M\}\,, \\ \Gamma_-^s = \{(p,d) \in \Gamma^-\,,\ p \in S\}\,. \end{cases}$$

Motivation for PINN method

PINN advantages

- Once the learning step is completed, the resolution is direct, and requires no additional time
- The ability of parallel processing
- Cost and Time Benefits for entreprise

Physical problem

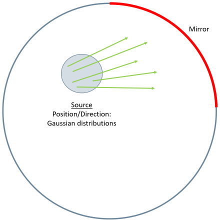

Figure: Schematic problem

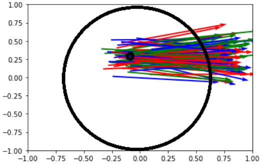

Figure: Rays shot

Question

Propose a PINN which makes it possible to approximate $f(p, d)$.

PINN Approach

General Loss formulation:

$$\mathcal{L} = \omega_{col}\mathcal{L}_{col} + \omega_{res}\mathcal{L}_{res} + \omega_{bc}\mathcal{L}_{bc} + \omega_{ic}\mathcal{L}_{ic}$$

Estimation

We propose a PINN estimating the density value $f(\theta, \varphi)$:

- PINN: MLP with 6 layers, TanH activation
- Loss: $\mathcal{L} = \omega_{col}\mathcal{L}_{col} + \omega_{bc}\mathcal{L}_{bc}$
- training set generated with statistical approach

Tests PINN

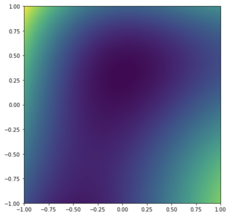

Figure: 1k epochs

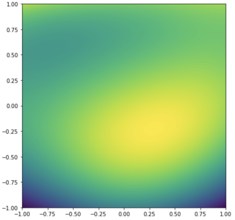

Figure: 2k epochs

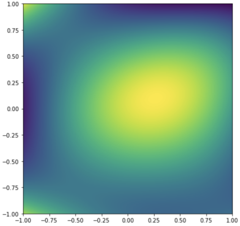

Figure: 20k epochs

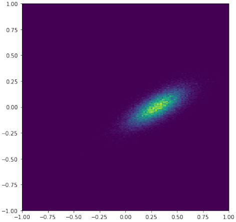

Figure: Groundtruth

Conclusion

Possible directions:

- Consider other activation functions and network structures
- If specific interest for surface values, possibility to take into account only those points
- For using residuals, consider the former equation and avoid trivial solution


Thank You!